Note abstract of “Posets whose persistence modules are always interval decomposable and homological invariants”

In one-parameter persistent homology analysis, it is a fundamental fact that every persistence module can be decomposed into *interval modules* due to Gabriel’s theorem, where each interval module is of the form $I[b,d]$ with $b \leqq c$. When dealing with multi-parameter settings, however, there are some difficulties with adapting the same techniques. Recently, there has been growing interest in the use of relative homological algebra to develop invariants using interval covers and interval resolutions (i.e., right minimal approximations and resolutions relative to interval-decomposable modules) for multi-parameter persistence modules.

In this talk, we study persistence modules over finite posets. Firstly, we provide another family of posets $C_{m,n}$ (which we call *commutative cycles*) indexed by two positive integers m,n satisfying the condition that every persistence module over it can be decomposed into interval modules. Conversely, we show that every poset satisfying that condition is either type A quiver or our commutative cycle. Similar to the one-parameter setting, one can consider the notion of persistence diagrams for the commutative cycle. Secondly, we show that over finite posets, the restriction of interval covers of persistence modules to each indecomposable direct summand is injective. This result suggests a way to simplify the computation of interval covers. This talk is based on joint work with Toshitaka Aoki and Emerson G. Escolar.